SHADE CALCULATIONS IN PHOTOVOLTAIC SYSTEMS

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Abstract - Shading of photovoltaic systems can cause high loss in performance. For the calculation of the performance loss the irradiance on each cell of the solar generator must be known. Then, the I-V-curve of a photovoltaic generator can be calculated using numerical methods. The irradiance on a tilted solar generator can be obtained from measurements of the global irradiance on the horizontal plane, geographical data and the calculated position of the sun. Objects, which possibly cause irradiance losses, are placed in the surroundings of the solar generator. The objects and the solar generator with each cell are suitable described by various plane surface polygons. Knowing the polygons and the position of the sun you can then calculate the reduced direct irradiance at each solar cell. Furthermore the reduced diffuse irradiance on the solar generator can be obtained by surface integrals. The addition of the reduced direct and diffuse irradiance and a ground reflection component leads to the reduced irradiance on the solar cells. Hence, the electrical performance of the solar generator can be calculated.

1. INTRODUCTION

Shading of photovoltaic generators can cause high loss in performance. If these losses are not taken into account, the power output of a photovoltaic system is often severely lower than expected. The performance losses should be calculated before setting up a photovoltaic system to avoid negative surprises. The I-V-curve and the performance of a solar module as well as of a solar generator can be calculated using numerical methods as proposed by Quaschning and Hanitsch (1995). To use these methods you must know the irradiance at each solar cell.
Perez et.al. (1987) described a method for calculating the irradiance on a tilted plane out of the irradiance on the horizontal plane. Furthermore the irradiance on the tilted plane can be reduced by objects in the surroundings of the solar generator. A method for calculating this reduced irradiance is described in this paper.

2. SHADING IN PHOTOVOLTAICS

The performance of homogeneously shaded photovoltaic generators decreases approximately proportionally to the irradiance reduction. In practice, photovoltaic generators are not always shaded homogeneously. On the contrary, shading at photovoltaic generators is non-uniform in most cases. In this case, the performance reduction is essentially higher than the mean irradiance reduction of the generator. Figure 1 shows the I-V curve of a photovoltaic module with 36 solar cells without bypass diodes. The first curve shows the well-known course of an unshaded module. The second curve was taken, when 75% of the first cell were shaded and the rest of the cells were fully irradiated. Although we only reduce the mean irradiance by about 2% the performance of the shaded photovoltaic module is 70% less. Furthermore the photovoltaic module can be damaged during shading, if too many cells are connected in series. To avoid cell damaging most of the module manufacturers install bypass diodes over cell strings up to 24 cells. The performance losses during shading can only be insignificantly reduced by these bypass diodes. Green (1985) proposed to integrate bypass diodes into all solar cells. This is one possibility of reducing performance losses at non-uniform illuminated photovoltaic generators. This idea, however, has not asserted itself.

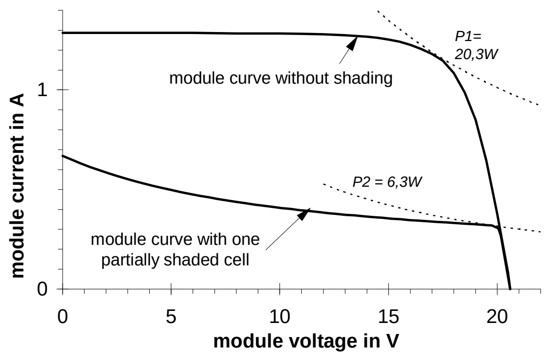

Fig. 1. I-V curves of an unshaded module SM50 without bypass diodes and the same module with one partially shaded cell (G=407 Wm^{-2}, T=300K).

We have shown that non-uniform shading of photovoltaic generators is a notable aspect. For calculating the electrical performance of a photovoltaic system, the reduced irradiance in the shading case is to be known for each cell. In the following sections we propose a suitable method for calculating the irradiance at the solar cells of a shaded photovoltaic generator.

3. SOLAR IRRADIANCE WITHOUT SHADING

3.1 *Irradiance on a horizontal plane*

The global irradiance G on the horizontal plane can be divided into the direct irradiance I and the diffuse irradiance D. For most sites in the world measurements of the global irradiance G are available for long periods of time. If no measurements of the diffuse irradiance D can be obtained for the same site, the global irradiance G can be split into direct and diffuse irradiance using equations based upon experimental studies of Liu and Jordan (1960) or Erbs, Klein and Duffie (1982).

3.2 *Irradiance on a tilted plane*

Usually, a photovoltaic generator is not set up horizontally. It can be tilted at the altitude angle α_M and it can be turned at the azimuth angle γ_M. Therefore the total solar irradiance G_t on the tilted photovoltaic generator surface is different from the global irradiance G on the horizontal plane. The total irradiance on the tilted plane is now composed of the direct irradiance I_t, the sky diffuse irradiance D_t , and another component, which is called ground reflection R_t.

$$G_t = I_t + D_t + R_t \tag{1}$$

The direct irradiance I_t on the tilted plane can be directly calculated from the direct irradiance I on the horizontal plane. To do this the position of the sun has to be known. It can be defined by two angles that are called sun altitude α_S and sun azimuth γ_S. These angles can be calculated from the longitude, latitude, date and time of the chosen location. One algorithm is described in DIN 5034 (1985), another algorithm is the modified SUNAE algorithm of Walraven (1978) (see also Wilkinson (1981, 1983) and Kambezidis and Papanikolaou (1990)).
The angle of incidence θ on the tilted plane can as well be calculated from the sun altitude and sun azimuth. The relationship between all angles is shown in figure 2.

$$\theta = \arccos(\cos\alpha_S \cdot \sin\alpha_M \cdot \cos(\gamma_S - \gamma_M) + \sin\alpha_S \cdot \cos\alpha_M) \tag{2}$$

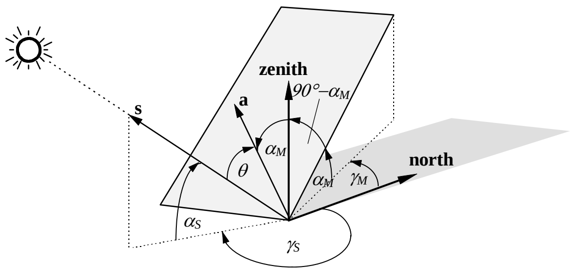

Fig. 2. Relationship between the angles at a tilted photovoltaic generator plane.

The direct irradiance on the tilted plane can now be calculated using the angle of incidence θ and the solar altitude α_S.

$$I_t = I \cdot \max(0, \frac{\cos\theta}{\sin\alpha_S}) \tag{3}$$

The ground reflection R_t can be calculated from the global irradiance G on the horizontal plane and the altitude angle of the tilted surface α_M using the following isotropic attachment.

$$R_t = G \cdot A \cdot \tfrac{1}{2} \cdot (1 - \cos\alpha_M) \tag{4}$$

If the albedo value A is not known, the value A = 0,2 should be chosen. Inneichen et. al. (1990) have shown that an unisotropic attachment gives only unessential improvements.
One method for calculating the sky diffuse irradiance on the tilted plane D_t was proposed by Klucher (1979). Utrillas and Martinez-Lozano (1994) have shown that the most exact algorithm for the calculation of the sky diffuse irradiance on the tilted plane was given by Perez et. al. (1987).
Before calculating the reduced irradiance on a tilted plane considering shading, we now propose a model for the surroundings of a photovoltaic generator.

4. MODELING SURROUNDINGS

First we introduce a rectangular coordinate system with the basis vector orientated to the north, east an zenith
Σ = {0, **n**, **e**, **z**}. The vector **s** pointing to the sun can be given as

$$\mathbf{s} = \begin{pmatrix} \cos\gamma_S \cdot \cos\alpha_S \\ \sin\gamma_S \cdot \cos\alpha_S \\ \sin\alpha_S \end{pmatrix} \tag{5}$$

All surfaces of the objects in the surroundings of the solar generator are given by planar polygons. All objects can consist of multiple surfaces, which are described by polygons with multiple vertices. Like that you can model regular shapes like cubes as well as irregular figures. Furthermore cylinders and spheres can approximately be described by polygon surfaces. All vertices can be given by a vector $\mathbf{o_i}$ in the **nez**-coordinate system. A cube for example consists of six surface polygons, each of them with four vertices.

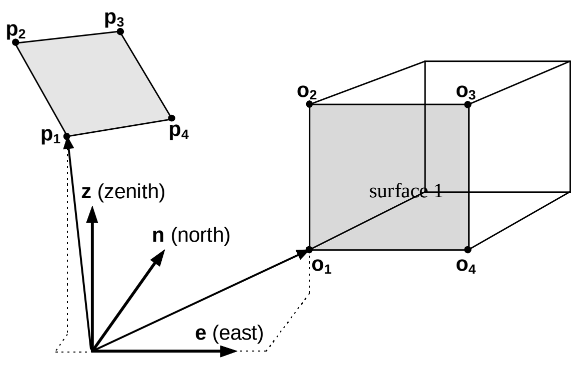

Fig. 3. Representation of a cube and a solar cell using polygons in the **nez**-coordinate system.

Irregular objects like trees can approximately be described by regular forms as spheres, cones and cylinders.
The photovoltaic generator, all photovoltaic modules, as well as all solar cells, are also described by planar

polygons. All vertices of these polygons are likewise described by a vector $\mathbf{p_i}$ in the **nez**-coordinate system.

5. REDUCED DIRECT IRRADIANCE

Shading of the direct irradiance occurs if an object is placed between the position of the sun and the solar generator surface. The shadow position of a single point $\mathbf{p_0}$ can easily be determined. Starting from the point $\mathbf{p_0}$ we have to search for the point of intersection $\mathbf{p_S}$ with the solar generator plane in the opposite direction of the vector **s**. Suppose the solar generator polygon is described by 4 vectors $\mathbf{p_1}..\mathbf{p_4}$, the vector $\mathbf{p_S}$ for the intersection between the straight in sun direction and the solar generator plane can be obtained by the following equation.

$$\mathbf{p_S} = \mathbf{p_0} - \frac{\mathbf{a} \cdot (\mathbf{p_0} - \mathbf{p_1})}{\mathbf{a} \cdot \mathbf{s}} \cdot \mathbf{s} \qquad (6)$$

The vector **a** is perpendicular to the solar generator plane and is given by

$$\mathbf{a} = (\mathbf{p_2} - \mathbf{p_1}) \times (\mathbf{p_4} - \mathbf{p_1}) \qquad (7)$$

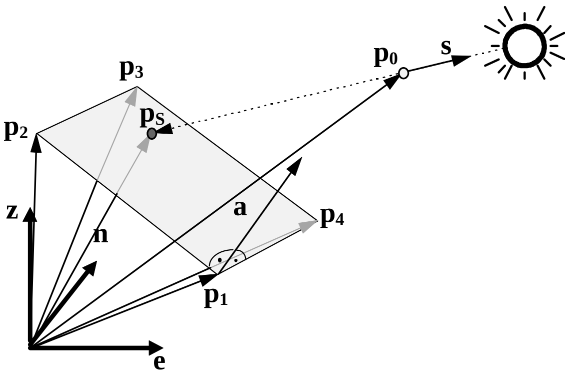

Fig. 4. Determination of the shadow position of a single point.

The shadow cast by the surface polygon of an object can be determined by calculating the point shade position $\mathbf{p_{Si}}$ of each vertex $\mathbf{o_i}$ of the surface polygon. All shade vertices also build a polygon that shall be called the shadow polygon of the surface polygon. For all surface polygons of an object the shadow polygons have to be calculated. All shadow polygons together build the shadow of the object. The shadow lies in the solar generator plane but it does not necessarily have to intersect the solar generator itself.

Only in rare cases the whole shadow is confined to the solar generator. In all other cases the shadow polygons have to be clipped to the solar generator polygon. This means that all parts of the shadow polygons, which are outside the solar generator polygon must be eliminated. To do this the polygon clipping algorithm of Sutherland and Hodgman (1974) can be used. The result is a shadow polygon that covers a part or the whole solar generator surface or a zero area polygon if the whole shadow is outside the solar generator. In the same manner the shadow polygons can be clipped to each photovoltaic module polygon as well as each solar cell polygon.

For the calculation of the reduced direct irradiance on a solar cell the total cell area A_C, as well as the shaded cell area A_S, has to be determined. Hence, we can introduce a direct irradiance reduction factor f_I

$$f_I = \frac{A_S}{A_C} \qquad (8)$$

The required areas A_C and A_S can be calculated from the solar cell polygon respectively the shadow polygons. Generally the polygon area of a plane polygon with n vertices $\mathbf{p_1}..\mathbf{p_n}$ can be obtained from

$$A = \frac{1}{2}\left|\sum_{i=1}^{n-2} (\mathbf{p_{i+1}} - \mathbf{p_1}) \times (\mathbf{p_{i+2}} - \mathbf{p_1})\right| \qquad (9)$$

If there are multiple shade polygons of multiple object surfaces the shade polygons can overlap. The shaded area is calculable from the sum of all shadow polygons if the overlapping areas are subtracted.

Another very simple but time consuming method for getting f_I is the graphical analysis. The solar cell polygon as well as all shadow polygons have to be drawn on the rastered screen in different colors. The areas are now rastered by the screen resolution.

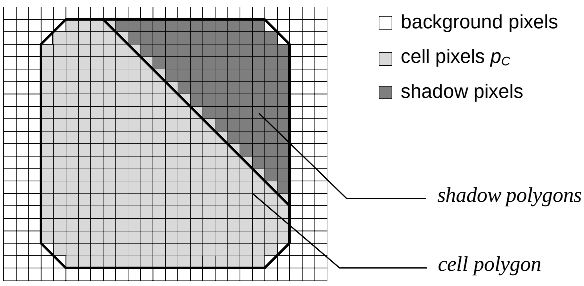

Fig. 5. Rastered shaded solar cell.

The direct irradiance reduction factor f_I can be obtained by counting the pixels in the cell color p_C and the pixels in the shade color p_S.

$$f_I = \frac{\sum_{y=1}^{y\,\max} \sum_{x=1}^{x\,\max} p_S(x,y)}{\sum_{y=1}^{y\,\max} \sum_{x=1}^{x\,\max} \left(p_C(x,y) + p_S(x,y)\right)} \qquad (10)$$

Finally, we obtain the reduced direct irradiance $I_{t,S}$

$$I_{t,S} = (1 - f_I) \cdot I_t \qquad (11)$$

6. REDUCED DIFFUSE IRRADIANCE

The diffuse irradiance is also reduced by objects around the solar generator. Unlike the direct irradiance, the diffuse irradiance has no defined direction. For the further calculation we suppose that the diffuse irradiance is constant over the whole solar generator. This is admissible if the objects are placed relatively far from the solar generator. In other cases the calculations for the reduced diffuse irradiance have to be repeated for several points of the solar generator.

The diffuse irradiance has to traverse a hypothetical hemisphere around the solar generator. All objects can be projected to this hemisphere. A coordinate transformation from the rectangular coordinate system $\Sigma = \{0, \mathbf{n}, \mathbf{e}, \mathbf{z}\}$ to the spherical coordinate system $\Sigma = \{0, \gamma, \alpha, \mathbf{r}\}$ has to be done. The radius r can be neglected. The object surface polygons can be drawn on the hemisphere.

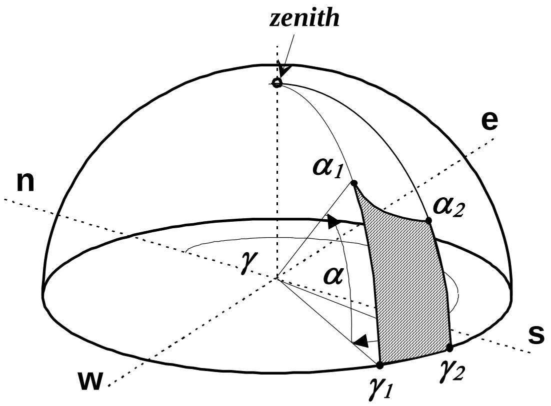

Fig. 6. A polygon projected on a hemisphere for calculating the diffuse irradiance reduction.

The diffuse irradiance on the tilted plane is reduced by the part of the diffuse irradiance that traverses the shaded area of the hemisphere. The diffuse irradiance reduction factor f_D can be calculated from the total diffuse irradiance on the tilted plane D_t and the amount of the diffuse irradiance, which traverses the shaded areas D_Δ of the hemisphere. The part of the hemisphere behind the solar generator has to be neglected before.

$$f_D = \frac{D_\Delta}{D_t} \qquad (12)$$

D_Δ can be obtained from a surface integration over the shaded areas on the hemisphere.

$$D_\Delta = \iint_{shade} L(\gamma,\alpha)\cos\theta\cos\alpha \cdot d\alpha d\gamma \qquad (13)$$

For an isotropic sky diffuse irradiance model $L(\gamma,\alpha)$ is constant for all sky directions. The diffuse irradiance reduction factor f_D can be written as

$$f_D = \frac{\iint_{shade} \cos\theta\cos\alpha \cdot d\alpha d\gamma}{\pi \cdot (1+\cos\alpha_M)} \qquad (14)$$

This factor is constant over the time and has to be determined only once. Normally an unisotropic diffuse sky model is used. One model for the calculation of L was proposed by Brunger and Eng (1989). Another model was given by Perez et.al. (1993). The calculations for the reduced diffuse irradiance become more exact with these models but also more time efficient. Then, the factor f_D is dependent on the sun position and the global irradiance.

Finally, the diffuse irradiance on a tilted and shaded plane $D_{t,S}$ is given by

$$D_{t,S} = (1 - f_D) \cdot D_t \qquad (15)$$

It is assumed that the ground reflection R_t is not reduced. A reduction of the ground reflection can be considered by a reduced albedo value.

7. CONCLUSIONS

We have shown, that shading at photovoltaic generators can cause high loss in performance. To estimate the performance of a solar generator surrounded with irradiance reducing objects the irradiance on each solar cell of the generator must be determined. The irradiance on the tilted solar generator plane can be obtained from measurements of the global irradiance at the horizontal plane. For the direct irradiance an irradiance reduction factor f_I was calculated for each cell. The diffuse irradiance reduction factor f_D was calculated once for the whole solar generator. Finally, the reduced irradiance for each solar cell of a tilted and shaded solar generator can be written as

$$G_{t,S} = (1 - f_I) I_t + (1 - f_D) \cdot D_t + R_t \qquad (16)$$

NOMENCLATURE

a	vector perpendicular to the solar module plane
A	albedo
A	area
A_C	solar cell area
A_S	shade area
D	diffuse irradiance on the horizontal plane
D_t	diffuse irradiance on the tilted plane
$D_{t,S}$	diffuse irradiance on the tilted, shaded plane
D_Δ	diffuse irradiance through the shaded part of the hemisphere
e	base vector to the east
f_D	diffuse irradiance reduction factor
f_I	direct irradiance reduction factor
G	global irradiance on the horizontal plane
G_t	total irradiance on the tilted plane
$G_{t,S}$	total irradiance on the tilted, shaded plane
i	index
I	direct irradiance on the horizontal plane
I	solar module current
I_t	direct irradiance on the tilted plane
$I_{t,S}$	direct irradiance on the tilted, shaded plane
L	sky luminance
n	number of vertices
n	base vector to the north
$\mathbf{o_i}$	vectors to the vertices of the object surface
P	solar module performance
p_C	pixel in cell color
$\mathbf{p_0}$	vector to an object point
$\mathbf{p_i}$	polygon vectors
$\mathbf{p_S}$	vector to the shade of an object point
p_S	pixel in shade color
R_t	ground reflection
s	vector to the sun position

T	temperature
V	solar module voltage
x	index
x_{max}	computer screen horizontal resolution
y	index
y_{max}	computer screen vertical resolution
$\mathbf{z}$	base vector to the zenith

Greek

α_M	solar module or solar generator altitude
α_S	sun altitude
γ_M	solar module or solar generator azimuth
γ_S	sun azimuth
θ	angle of incidence

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